PhD VIVA

Sequential Bayesian Learning for State Space Models

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March 17, 2023

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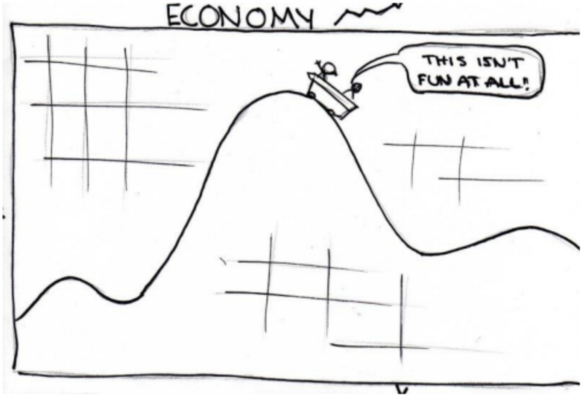
Motivation

Economies move in cycles



Credit: *Economics fun*.

HSMM Project

Overview

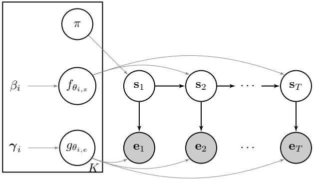
- Working Title:
  Sequential Bayesian Learning for Hidden Semi-Markov Models
- Authorship: Jointly co-authored with Dr. Konstantinos Kalogeropoulos
- Timeline: First project

Motivation

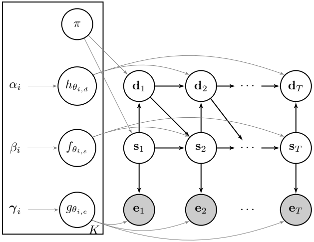
- Discrete State Space Models (SSM) have applications in: ecology, economics, finance, robotics and signal processing *.
- Most popular SSM known as Hidden Markov Models (HMM)
- Hidden semi-Markov Models (HSMM) a more flexible HMM extension
- Goal: provide methods to estimate HSMM in
  - (1) a computationally feasible time,
  - (2) an exact manner, i.e. only subject to Monte Carlo error,
  - (3) a sequential setting.

(*) (Bulla and Bulla, 2006; Lindsten and Schön, 2013; Chopin and Papaspiliopoulos, 2020; Corenflos et al., 2021)

Model Setup - From HMM to HSMM



(a)



(b)

(a) HMM with observed data $e_t \sim g_\theta(e_t \mid s_t)$ and latent state $s_t \sim f_\theta(s_t \mid s_{t-1})$.

(a) $p(s_{t+k} = j,\ s_{t+1:t+k-1} = i \mid s_t = i)$ is implicitly geometric.

(b) HSMM extends model via latent $d_t \sim \begin{cases} \delta(d_t, d_{t-1} - 1) & d_{t-1} > 0 \\ h_\theta(d_t \mid s_t, d_{t-1}) & d_{t-1} = 0 \end{cases}$,

which governs the state duration for $s_t \sim \begin{cases} \delta(s_t, s_{t-1}) & d_{t-1} > 0 \\ f_\theta(s_t \mid s_{t-1}, d_{t-1}) & d_{t-1} = 0 \end{cases}$

Challenges

- Computational complexity for HMM likelihood: $\mathcal{O}(K^2 T)$*, where K = number of latent states, T = number of data points.
- Computational complexity for HSMM likelihood: $\mathcal{O}(K^2 (d_{max} - d_{min})^2 T)$**, where d_{min} and d_{max} denote the minimal and maximal state duration in a latent regime.
- Idea: Use a particle filter for the likelihood computation in $\mathcal{O}(NT)$ operations, where N denotes the number of particles used in the filter.

(*) (Baum and Petrie, 1966)
(**) (Murphy, 2002; Dewar et al., 2012)

Contribution

- We provide and verify an efficient computational scheme for Bayesian parameter estimation on HSMMs.
- We demonstrate how this algorithm can be used for regime switching, model selection and clustering purposes.
- We propose a novel class of models by linking AR-type models with HSMMs to better describe data consisting of financial time series.

Covid-19 Project

Overview

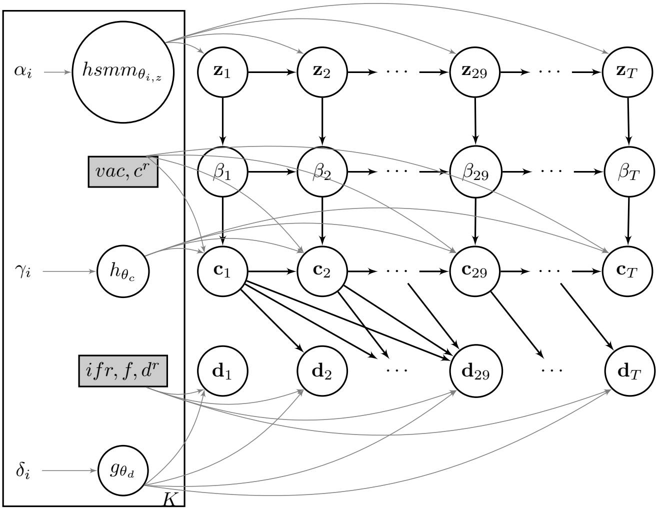
- Working Title:
  SIR-type State Space Models with Piecewise Constant Transmission Rates
- Authorship: Jointly co-authored with Dr. Konstantinos Kalogeropoulos and Dr. Nikolaos Demiris
- Timeline: Second project

Motivation

- Standard techniques to model SARS-CoV-2 like phenomena are known as compartmental models (SIR, SEIR, etc.)
- Important factor: $R_t = \frac{\beta S(t)}{\gamma N(t)}$, the number of secondary infections that one infected person would produce through the entire duration of the infectious period.
- Typically, β, the transmission rate between susceptible and infected individuals, is kept constant across the time horizon.
- Goal: Make β time varying, but keep ease of interpretability for the model.

(*) (Cohen, 1992; Diekmann et al., 2012)
(**) (Flaxman et al., 2020)

Model Setup - HSMM style Epidemic Model



Model Setup - HSMM style Epidemic Model

- Reported cases $c_t^r \sim \text{Negative Binomial}_{Alternative}\left(c_t^*, c_t^* + \frac{c_t^{*2}}{\phi_c}\right)$, where c_t^* are the model implied cases
- Reported deaths $d_t^r \sim \text{Negative Binomial}_{Alternative}\left(d_t^i, d_t^i + \frac{d_t^{i2}}{\phi_d}\right)$ where d_t^i are the model implied deaths.
- d_t^i are a function c_t^*, which are obtained from solving the ODE

$$
\begin{aligned}
\frac{\mathrm{d}S_t}{\mathrm{d}t} &= -\beta_t S_t \frac{(I_{1,t} + I_{2,t})}{N} - \rho\nu_{t-U}, \\
\frac{\mathrm{d}E_{1,t}}{\mathrm{d}t} &= \beta_t S_t \frac{(I_{1,t} + I_{2,t})}{N} - \epsilon E_{1,t}, \\
\frac{\mathrm{d}E_{2,t}}{\mathrm{d}t} &= \epsilon E_{1,t} - \epsilon E_{2,t}, \\
\frac{\mathrm{d}I_{1,t}}{\mathrm{d}t} &= \epsilon E_{2,t} - \gamma I_{1,t}, \\
\frac{\mathrm{d}I_{2,t}}{\mathrm{d}t} &= \gamma I_{1,t} - \gamma I_{2,t}, \\
\frac{\mathrm{d}R_t}{\mathrm{d}t} &= \gamma I_{2,t} + \rho\nu_{t-U},
\end{aligned}
$$

Challenges

- Attaching latent state sequence to β significantly increases computational complexity for likelihood.
- Observation model significantly more computational complexity than previous project.
- Significant expert knowledge required for model parameter.
- Underlying data very noisy.

Contribution

- Proposed a novel HSMM-Epidemic Model (HSMMEM) that is more flexible than its alternatives but still convenient to interpret.
- Designed a tailored Particle Filter (PF) that can be used efficiently for sequential inference.
- Identified the number of latent regimes for the provided data.
- Showed that a combining reported fatalities and infections enhances predictive performance.

SV Copula Project

Overview

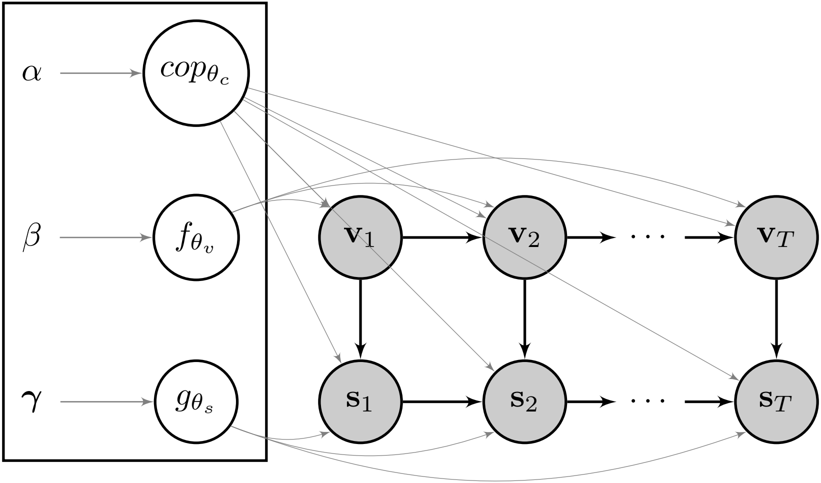
- Working Title:
  A Class of Stochastic Volatility Models with Copula Dependencies
- Authorship: Jointly co-authored with Dr. Konstantinos Kalogeropoulos, Prof. Alexandros Beskos and Dr. Aristidis Nikolopoulos
- Timeline: Third project

Motivation

- Stochastic Volatility (SV) models can capture important stylised effects *.
- Empirically, joint distribution of prices and volatilities has asymmetric structure**.
- Copula theory provides an flexible modelling framework for this case ***.
- Goal: Incorporate Copula dependencies and suggest useful models for log-prices and volatilities.

(*) (Ghysels et al., 1996; Shephard, 1996)
(**) (Ning et al., 2008)
(***) (Joe, 2014)

Model Setup - Stochastic Volatility Copula



Model Setup - Stochastic Volatility Copula

- Discretization of Heston Model based on Euler-Maruyama scheme:

$$S_t = S_{t-1} + \big(\mu_S - \exp(X_t)/2\big)\delta + \sqrt{\delta}\,\exp(X_t/2)\,\epsilon_t,$$

$$X_t = X_{t-1} + \frac{\kappa\big(\mu_V - \exp(X_{t-1})\big) - \frac{1}{2}\sigma^2}{\exp(X_{t-1})}\delta + \sigma\sqrt{\delta}\exp(-X_{t-1}/2)\,\zeta_t\ .$$

- Error term dependency structure typically specified as

$$(\epsilon_i, \zeta_i) \sim N\Big(0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\Big),$$

where $\rho \in (-1, 1)$ is known as leverage effect.

- We propose a more flexible dependency structure using the Copula machinery. For marginals F_S and F_V, the transformed noises are:

$$(\tilde{\epsilon}_{i_i}, \tilde{\zeta}_i) := (F_S(\epsilon_i), F_V(\zeta_i)) \sim C(\tilde{\epsilon}_{i_i}, \tilde{\zeta}_i; \theta)$$

Challenges

- Identify a suitable set of Copula choices
- Does any of those perform better versus the benchmark Normal Copula?

Contribution

- We explored a new class of Stochastic Volatility Models with Copula dependencies.
- We benchmarked carefully selected Copula choices based on different model comparison criteria and their predictive performance.
- The Frank copula outperformed standard choices in financial modelling literature with respect to all benchmarks and across all time horizons.

Discussion

Appendix

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